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LONG PERIOD EFFECTS OF THE ELLIPTICITY OF THE EARTH'S EQUATOR ON THE MOTION OF ARTIFICIAL SATELLITES

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SUMMARY

If the mean daily motion of a satellite is nearly commensurable with the angular velocity of rotation of the earth, long period perturbations will influence the motion of the satellite. The eccentricity of such a commensurable orbit is not necessarily small, as the example of Explorer VI shows. This circumstance has brought about the development of a semi-analytical method for treating the perturbations. This method avoids the need to develop the disturbing function into powers of the eccentricity; consequently, it includes the case of elongated orbits as well as those with moderate eccentricities.

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LONG PERIOD EFFECTS OF THE ELLIPTICITY OF THE EARTH'S EQUATOR ON THE MOTION OF ARTIFICIAL SATELLITES*

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INTRODUCTION

This article considers the long period perturbations in the motion of a satellite, produced by the ellipticity of the earth's equator. If the mean daily motion of the satellite is nearly commensurable with the angular velocity of rotation of the earth, then long period (critical) terms appear in the development of the disturbing function. There are methods, developed by Sehnal (Reference 1), Cook (Reference 2), and O'Keefe and Batchlor (Reference 3), which are applicable to cases in which the eccentricity is small and the coefficients of the critical terms can be developed into rapidly convergent power series in the eccentricity. However, eccentricities of the commensurable orbits are not necessarily small, as Explorer VI (1959 δ) shows. Furthermore, in many cases, such as $n'/n = 2/3$ or $2/5$, where n is the satellite's mean motion and n' is the earth's angular velocity of rotation, the influence of the ellipticity of the equator can be more easily found from measurements made when the eccentricity is not small. These circumstances brought about the development of the theory presented herein, which is applicable to moderate and large eccentricities indiscriminately, up to approximately the point where the eccentricity, e, of the orbit is 0.8. This value can be considered the upper limit for practical application of the theory, because for a larger e the basic series of the theory begins to converge more slowly. The basic idea of this method is similar to the idea used by Brouwer (Reference 4) in treating the secular perturbations of eccentric orbits.

DEVELOPMENT OF THE DISTURBING FUNCTION

In this method instead of developing the coefficients of the critical terms in the disturbing function and its derivatives into series in powers of the eccentricity, one

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substitutes the numerical value of e at the outset and computes the values of the coefficients by means of numerical integration. Thus, the development is numerical with respect to the eccentricity, but literal with respect to the remaining elements. Satellite motion is referred to the system of coordinates rigidly connected with the rotating earth. The coordinates of the satellite (x, y, z) are given with respect to this system, but the elliptic elements of the satellite's orbit are given with respect to the inertial system having its origin in the center of the earth, the x and y axes lying in the equator and the z axis directed toward the north pole. With the proper choice of the moving axes (Reference 3) the disturbing function Ω can be represented in the form:

$$\Omega = \frac{A_{22}}{r^3}\left(\frac{x^2 - y^2}{r^2}\right), \tag{1}$$

where r is the radius vector of the satellite, and

$$\begin{aligned} x &= r\cos(f+g)\cos(h-n't) - r\sin(f+g)\sin(h-n't)\cos i, \\ y &= r\cos(f+g)\sin(h-n't) + r\sin(f+g)\cos(h-n't)\cos i, \\ z &= r\sin(f+g)\sin i. \end{aligned} \tag{2}$$

In Equation 2, f is the true anomaly of the satellite, g is the argument of perigee, h is the right ascension of the ascending node, t is time, and i is the inclination of the orbit with respect to the equator. Substituting Equation 2 into Equation 1 we deduce:

$$\begin{aligned} \Omega &= \frac{A_{22}}{4a^3}(1+\cos i)^2\left(\frac{a}{r}\right)^3\cos(2f+2g+2h-2n't) \\ &+ \frac{A_{22}}{4a^3}(1-\cos i)^2\left(\frac{a}{r}\right)^3\cos(2f+2g-2h+2n't) \\ &+ \frac{A_{22}}{2a^3}\sin^2 i\left(\frac{a}{r}\right)^3\cos(2h-2n't), \end{aligned} \tag{3}$$

where a is the semi-major axis of the orbit of the satellite. When the critical argument $w = sl - 2n't$ is introduced, l being the mean anomaly of the satellite and

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$n/n' \approx 2/s$, we have, substituting $sl - w$ for $2n't$ into Equation 3,

$$\Omega = \frac{A_{22}}{4a^3}(1+\cos i)^2\left(\frac{a}{r}\right)^3\cos(2f - sl + w + 2g + 2h)$$

$$+\frac{A_{22}}{4a^3}(1-\cos i)^2\left(\frac{a}{r}\right)^3\cos(2f + sl - w + 2g - 2h)$$

$$+\frac{A_{22}}{2a^3}\sin^2 i\left(\frac{a}{r}\right)^3\cos(-sl + w + 2h)\,. \tag{4}$$

By using Equation 4, the long period part of the disturbing function may be written:

$$R = \frac{1}{2\pi}\int_0^{2\pi}\Omega\, dl\,. \tag{5}$$

Taking into consideration

$$\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{a}{r}\right)^p \sin(qf - sl)\, dl = 0\,,$$

and defining

$$Q_s^{p,q} = \frac{1}{\pi}\int_0^{\pi}\left(\frac{a}{r}\right)^p\cos(qf - sl)\, dl\,, \tag{6}$$

we deduce:

$$R = \frac{A_{22}}{4a^3}(1+\cos i)^2 Q_s^{3,2}\cos(sl + 2g + 2h - 2n't)$$

$$+\frac{A_{22}}{4a^3}(1-\cos i)^2 Q_{-s}^{3,2}\cos(sl - 2g + 2h - 2n't)$$

$$+\frac{A_{22}}{2a^3}\sin^2 i\, Q_s^{3,0}\cos(sl + 2h - 2n't)\,. \tag{7}$$

The coefficients $Q_s^{p,q}$ are functions only of the eccentricity and could be computed by numerical integration for a given value of e. However, if the eccentricity is large, neither the mean, the eccentric, nor the true anomaly is convenient to use as the basic variable in the process of integration along the whole orbit. In some part of the orbit one of the anomalies might be more convenient than another, but the use of any one anomaly along the complete orbit leads to the multiplication of a large number by a small one. In addition, the use of Kepler's equation becomes inconvenient. For these reasons a new anomaly u is introduced by means of the equation

$$\frac{1}{2} f = \mathrm{am}(u, k), \tag{8}$$

where

$$k^2 = \frac{2e}{1+e} \tag{9}$$

and the complementary modulus k' is obtained from

$$k'^2 = \frac{1-e}{1+e} . \tag{10}$$

Introduction of u leads to the representation of the integrand by a rapidly convergent series, even for large eccentricities; the troublesome divisor $1 - e$ will appear in front of the integral. Basically, the anomaly u does not differ from the anomaly introduced by Gravelius (Reference 5) in treating planetary perturbations for the case of highly eccentric orbits. The Gravelius transformation is a special case of Glydén's transformation (Reference 6). We deduce from Equation 8 that

$$\frac{a}{r} = \frac{1 + e \cos f}{1 - e^2} = \frac{1 + k'^2}{2k'^2} \mathrm{dn}^2 u, \tag{11}$$

and from Equations 8 and 11 we find that

$$d\epsilon = \frac{1}{\sqrt{1-e^2}} \cdot \frac{r}{a} df = \frac{2k'}{\mathrm{dn}\, u} du \tag{12}$$

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and

$$dl = 2\frac{r}{a} \cdot \frac{k'}{\mathrm{dn}\, u}\, du\,, \tag{13}$$

where ϵ is the eccentric anomaly of the satellite. It follows from Equations 11 and 13 that

$$dl = \frac{4k'^{3}}{1 + k'^{2}} \cdot \frac{du}{\mathrm{dn}^{3}\, u}\;. \tag{14}$$

Taking

$$\frac{k'}{\mathrm{dn}\, u} = \mathrm{dn}\,(u + K)$$

and

$$\mathrm{dn}^{3}\, u = \frac{1 + k'^{2}}{2}\,\mathrm{dn}\, u - \frac{1}{2}\frac{d^{2}}{du^{2}}\,(\mathrm{dn}\, u)$$

into account, we have from Equation 14:

$$dl = 2\,\mathrm{dn}\,(u + K)\, du - \frac{2}{1 + k'^{2}}\,\frac{d^{2}}{du^{2}}\,\mathrm{dn}\,(u + K)\, du$$

and therefore,

$$l = 2\,\mathrm{am}\,(u + K) - \pi - \frac{2}{1 + k'^{2}}\,\frac{d}{du}\frac{k'}{\mathrm{dn}\, u}\;. \tag{15}$$

The value for $2K$ is computed using the formulas

$$P = \frac{1 - \sqrt{k'}}{1 + \sqrt{k'}} ,$$

$$\lambda = \left(\frac{P}{2}\right) + 2\left(\frac{P}{2}\right)^5 + 15\left(\frac{P}{2}\right)^9 + \cdots\cdots ,$$

and

$$\sqrt{\frac{2K}{\pi}} = 1 + 2\lambda + 2\lambda^4 + 2\lambda^9 + \cdots\cdots$$

We obtain, using Equations 6, 8, 11, 14, and 15:

$$Q_s^{p,q} = \frac{1}{2^{p-2}} \frac{\left(1 + k'^2\right)^{p-1}}{k'^{2p-3}} \frac{1}{\pi} \int_0^K \cos\left(qf - sl\right) \cdot dn^{2p-3} u \, du \tag{16}$$

and

$$qf - sl = 2q \text{ am } u - 2s \text{ am } (u + K) + s\pi + \frac{2s}{1 + k'^2} \frac{d}{du} \frac{k'}{dn\ u} . \tag{17}$$

The argument $qf - sl$ and the value of $dn^{2p-3}\,u$ can be developed into fast convergent Fourier series, and only the first few terms of these series need be taken into consideration to compute the perturbations. Taking the developments

$$\text{am } u = \frac{\pi u}{2K} + \sum_{m=1}^{\infty} \frac{2\lambda^m}{m\left(1 + \lambda^{2m}\right)} \sin\frac{m\pi u}{K} ,$$

$$\frac{k'}{dn\ u} = \frac{\pi}{2K} + \frac{\pi}{K} \sum_{m=1}^{\infty} (-1)^m \frac{2\lambda^m}{1 + \lambda^{2m}} \cos\frac{m\pi u}{K} , \tag{18}$$

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into consideration, we deduce from Equations 15 and 17 the following rapidly convergent series:

$$l = \frac{\pi u}{K} + 4 \sum_{m=1}^{\infty} (-1)^m \left(\frac{1}{m} + \frac{\pi^2}{K^2} \frac{m}{1 + k'^2}\right) \left(\frac{\lambda^m}{1 + \lambda^{2m}}\right) \sin \frac{m\pi u}{K} ; \tag{19}$$

$$qf - sl = \frac{\pi(q - s)}{K} u$$

$$+ 4 \sum_{m=1}^{\infty} \left[\frac{q + s(-1)^{m+1}}{m} + \frac{\pi^2}{K^2} \frac{ms(-1)^{m+1}}{1 + k'^2}\right] \frac{\lambda^m}{1 + \lambda^{2m}} \cdot \sin \frac{m\pi u}{K} . \tag{20}$$

In addition, we find

$$\mathrm{dn}\ u = \frac{\pi}{2K} + \frac{\pi}{K} \sum_{m=1}^{\infty} \frac{2\lambda^m}{1 + \lambda^{2m}} \cos \frac{m\pi u}{K} . \tag{21}$$

By substituting the numerical values of Equations 20 and 21 into Equation 16 the values of $Q_s^{p,q}$ can easily be obtained by means of a numerical quadrature. The value of λ is small even for large eccentricities, for example, for $e = 0.8$, $\lambda = 0.134$.

In order to compute the perturbations of the elements we must form the derivatives of R with respect to the elements. All the derivatives can be easily computed analytically except the derivative with respect to the eccentricity, which must be computed numerically. Differentiating Equation 6 with respect to e and taking

$$\frac{\partial}{\partial e}\left(\frac{a}{r}\right) = \frac{a^2}{r^2} \cos f$$

and

$$\frac{\partial f}{\partial e} = \left(\frac{a}{r} + \frac{1}{1 - e^2}\right) \sin f$$

into consideration, we obtain a general formula for the numerical computation of the derivative of $Q_s^{p,q}$ with respect to e:

$$\frac{\partial}{\partial e}\left(Q_s^{p,q}\right) = \frac{p + q}{2}\left(Q_s^{p+1,q+1}\right) + \frac{p - q}{2}\left(Q_s^{p+1,q-1}\right) + \frac{q}{2}\left(\frac{1}{1 - e^2}\right)\left(Q_s^{p,q+1} - Q_s^{p,q-1}\right). \tag{22}$$

In our particular case we have, taking $Q_s^{p,q} = Q_{-s}^{p,-q}$ into consideration,

$$\frac{\partial}{\partial e}\left(Q_s^{3,2}\right) = \frac{5}{2} Q_s^{4,3} + \frac{1}{2} Q_s^{4,1} + \frac{1}{1-e^2}\left(Q_s^{3,3} - Q_s^{3,1}\right)$$

and

$$\frac{\partial}{\partial e}\left(Q_s^{3,0}\right) = \frac{3}{2}\left(Q_s^{4,1} + Q_{-s}^{4,1}\right) .$$

THE PROBLEM OF INTEGRATION

In performing the integration the canonical elements of Delaunay are used. This choice is justified by the simplicity of the process by which the perturbations can be obtained (References 7 and 8). It is assumed that the commensurability is not extremely sharp and that the integrals can be represented in a trigonometrical form. If there is a sharp commensurability the theory of resonance should be applied. In terms of Delaunay canonical variables,

$$L = \sqrt{a}, \qquad l,$$

$$G = \sqrt{a\left(1 - e^2\right)}, \qquad g,$$

$$H = \sqrt{a\left(1 - e^2\right)} \cos i, \qquad h,$$

the disturbing function can be expressed in the form

$$\begin{aligned} R = {} & \frac{1}{4} \frac{A_{22}}{L^6}\left(1 + \frac{H}{G}\right)^2 Q_s^{3,2} \cos\left(sl + 2g + 2h - 2n't\right) \\ & + \frac{1}{4} \frac{A_{22}}{L^6}\left(1 - \frac{H}{G}\right)^2 Q_{-s}^{3,2} \cos\left(sl - 2g + 2h - 2n't\right) \\ & + \frac{1}{2} \frac{A_{22}}{L^6}\left(1 - \frac{H^2}{G^2}\right) Q_s^{3,0} \cos\left(sl + 2h - 2n't\right) . \end{aligned} \tag{23}$$

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The gravitational constant is set equal to one. The value of A_{22} is comparable to the value of the coefficient of the fourth harmonic. The perturbations will be limited herein to those of the first order with respect to A_{22}. The mean values of l, g, and h can be taken from Brouwer (Reference 9), and after the integration is performed L, G, and H can be replaced by their "undisturbed" values. We deduce from:

$$\frac{dL}{dt} = \frac{\partial R}{\partial l}, \qquad \frac{dl}{dt} = -\frac{\partial R}{\partial L};$$

$$\frac{dG}{dt} = \frac{\partial R}{\partial g}, \qquad \frac{dg}{dt} = -\frac{\partial R}{\partial G};$$

$$\frac{dH}{dt} = \frac{\partial R}{\partial h}, \qquad \frac{dh}{dt} = -\frac{\partial R}{\partial H};$$

the following equations:

$$\begin{aligned}\frac{dL}{dt} = & -\frac{1}{4}\frac{A_{22}}{L^6}\, s\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\sin\left(sl + 2g + 2h - 2n't\right)\\ & -\frac{1}{4}\frac{A_{22}}{L^6}\, s\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\sin\left(sl - 2g + 2h - 2n't\right)\\ & -\frac{1}{2}\frac{A_{22}}{L^6}\, s\left(1-\frac{H^2}{G^2}\right) Q_s^{3,0}\sin\left(sl + 2h - 2n't\right);\end{aligned} \tag{24}$$

$$\begin{aligned}\frac{dG}{dt} = & -\frac{1}{2}\frac{A_{22}}{L^6}\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\ \sin\left(sl + 2g + 2h - 2n't\right)\\ & +\frac{1}{2}\frac{A_{22}}{L^6}\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\ \sin\left(sl - 2g + 2h - 2n't\right);\end{aligned} \tag{25}$$

$$\frac{dH}{dt} = -\frac{1}{2}\frac{A_{22}}{L^6}\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\sin\left(sl + 2g + 2h - 2n't\right)$$

$$-\frac{1}{2}\frac{A_{22}}{L^6}\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\sin\left(sl - 2g + 2h - 2n't\right)$$

$$-\frac{A_{22}}{L^6}\left(1-\frac{H^2}{G^2}\right) Q_s^{3,0}\sin\left(sl + 2h - 2n't\right). \tag{26}$$

Taking

$$\frac{\partial Q_s^{p,q}}{\partial L} = \frac{1}{e}\left(\frac{G^2}{L^3}\right)\frac{\partial Q_s^{p,q}}{\partial e}$$

and

$$\frac{\partial Q_s^{p,q}}{\partial G} = -\frac{1}{e}\left(\frac{G}{L^2}\right)\frac{\partial Q_s^{p,q}}{\partial e}$$

into account:

$$\frac{dl}{dt} = -\frac{\partial R}{\partial L} - \frac{1}{e}\left(\frac{G^2}{L^3}\right)\frac{\partial R}{\partial e}$$

$$= \frac{A_{22}}{L^7}\left[\frac{3}{2}Q_s^{3,2} - \frac{1}{4e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_s^{3,2}}{\partial e}\right]\left(1+\frac{H}{G}\right)^2\cos\left(sl + 2g + 2h - 2n't\right)$$

$$+\frac{A_{22}}{L^7}\left[\frac{3}{2}Q_{-s}^{3,2} - \frac{1}{4e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_{-s}^{3,2}}{\partial e}\right]\left(1-\frac{H}{G}\right)^2\cos\left(sl - 2g + 2h - 2n't\right)$$

$$+\frac{A_{22}}{L^7}\left[3Q_s^{3,0} - \frac{1}{2e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_s^{3,0}}{\partial e}\right]\left(1-\frac{H^2}{G^2}\right)\cos\left(sl + 2h - 2n't\right)\ ; \tag{27}$$

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$$\frac{dg}{dt} = -\frac{\partial R}{\partial G} + \frac{1}{e}\left(\frac{G}{L^2}\right)\frac{\partial R}{\partial e}$$

$$= \frac{1}{4}\frac{A_{22}}{L^7}\left[2\,\frac{H}{L}\left(\frac{L^2}{G^2}\right)\left(1+\frac{H}{G}\right)Q_s^{3,2} + \left(1+\frac{H}{G}\right)^2 \frac{1}{e}\,\frac{\partial Q_s^{3,2}}{\partial e}\left(\frac{G}{L}\right)\right]\cos\left(sl + 2g + 2h - 2n't\right)$$

$$-\frac{1}{4}\frac{A_{22}}{L^7}\left[2\,\frac{H}{L}\left(\frac{L^2}{G^2}\right)\left(1-\frac{H}{G}\right)Q_{-s}^{3,2} - \left(1-\frac{H}{G}\right)^2 \frac{1}{e}\,\frac{\partial Q_{-s}^{3,2}}{\partial e}\left(\frac{G}{L}\right)\right]\cos\left(sl - 2g + 2h - 2n't\right)$$

$$-\frac{1}{2}\frac{A_{22}}{L^7}\left[2\,\frac{H^2}{L^2}\cdot\frac{L^3}{G^3}\,Q_s^{3,0} - \left(1-\frac{H^2}{G^2}\right)\frac{1}{e}\,\frac{\partial Q_s^{3,0}}{\partial e}\left(\frac{G}{L}\right)\right]\cos\left(sl + 2h - 2n't\right), \tag{28}$$

and

$$\frac{dh}{dt} = -\frac{1}{2}\frac{A_{22}}{L^7}\left(1+\frac{H}{G}\right)\left(\frac{L}{G}\right)Q_s^{3,2}\cos\left(sl + 2g + 2h - 2n't\right)$$

$$+\frac{1}{2}\frac{A_{22}}{L^7}\left(1-\frac{H}{G}\right)\left(\frac{L}{G}\right)Q_{-s}^{3,2}\cos\left(sl - 2g + 2h - 2n't\right)$$

$$+\frac{A_{22}}{L^7}\left(\frac{H}{L}\right)\left(\frac{L^2}{G^2}\right)Q_s^{3,0}\cos\left(sl + 2h - 2n't\right). \tag{29}$$

Putting

$$\gamma_{22} = \frac{A_{22}}{L^4},$$

and designating the mean motions of the arguments $sl + 2g + 2h - 2n't$ and $sl - 2g + 2h - 2n't$ and $sl + 2h - 2n't$, respectively, by $\frac{W_1}{L^3}$, $\frac{W_2}{L^3}$, and $\frac{W_3}{L^3}$, we deduce the following expressions for the perturbations:

$$\frac{\delta L}{L} = \frac{1}{4}\frac{s\gamma_{22}}{W_1}\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\cos\left(sl + 2g + 2h - 2n't\right)$$

$$+\frac{1}{4}\frac{s\gamma_{22}}{W_2}\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\cos\left(sl - 2g + 2h - 2n't\right)$$

$$+\frac{1}{2}\frac{s\gamma_{22}}{W_3}\left(1-\frac{H^2}{G^2}\right) Q_s^{3,0}\cos\left(sl + 2h - 2n't\right) ; \qquad (30)$$

$$\frac{\delta G}{L} = \frac{\gamma_{22}}{2W_1}\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\cos\left(sl + 2g + 2h - 2n't\right)$$

$$-\frac{\gamma_{22}}{2W_2}\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\cos\left(sl - 2g + 2h - 2n't\right) ; \qquad (31)$$

$$\frac{\delta H}{L} = \frac{\gamma_{22}}{2W_1}\left(1+\frac{H}{G}\right)^2 Q_s^{3,2}\cos\left(sl + 2g + 2h - 2n't\right)$$

$$+\frac{\gamma_{22}}{2W_2}\left(1-\frac{H}{G}\right)^2 Q_{-s}^{3,2}\cos\left(sl - 2g + 2h - 2n't\right)$$

$$+\frac{\gamma_{22}}{W_3}\left(1-\frac{H^2}{G^2}\right)\cos\left(sl + 2h - 2n't\right) ; \qquad (32)$$

$$\delta l = \frac{\gamma_{22}}{W_1}\left[\frac{3}{2}Q_s^{3,2} - \frac{1}{4e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_s^{3,2}}{\partial e}\right]\left(1+\frac{H}{G}\right)^2\sin\left(sl + 2g + 2h - 2n't\right)$$

$$+\frac{\gamma_{22}}{W_2}\left[\frac{3}{2}Q_{-s}^{3,2} - \frac{1}{4e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_{-s}^{3,2}}{\partial e}\right]\left(1-\frac{H}{G}\right)^2\sin\left(sl - 2g + 2h - 2n't\right)$$

$$+\frac{\gamma_{22}}{W_3}\left[3Q_s^{3,0} - \frac{1}{2e}\left(\frac{G^2}{L^2}\right)\frac{\partial Q_s^{3,0}}{\partial e}\right]\left(1-\frac{H^2}{G^2}\right)\sin\left(sl + 2h - 2n't\right); \qquad (33)$$

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$$\delta g = \frac{1}{4}\frac{\gamma_{22}}{W_1}\left[2\frac{H}{L}\left(\frac{L^2}{G^2}\right)\left(1+\frac{H}{G}\right)Q_s^{3,2} + \frac{1}{e}\left(1+\frac{H}{G}\right)^2\left(\frac{G}{L}\right)\frac{\partial Q_s^{3,2}}{\partial e}\right]\sin(sl + 2g + 2h - 2n't)$$

$$-\frac{1}{4}\frac{\gamma_{22}}{W_2}\left[2\frac{H}{L}\left(\frac{L^2}{G^2}\right)\left(1-\frac{H}{G}\right)Q_{-s}^{3,2} - \frac{1}{e}\left(1-\frac{H}{G}\right)^2\left(\frac{G}{L}\right)\frac{\partial Q_{-s}^{3,2}}{\partial e}\right]\sin(sl - 2g + 2h - 2n't)$$

$$-\frac{1}{2}\frac{\gamma_{22}}{W_3}\left[2\frac{H^2}{L^2}\left(\frac{L^3}{G^3}\right)Q_s^{3,0} - \left(1-\frac{H^2}{G^2}\right)\frac{1}{e}\left(\frac{G}{L}\right)\frac{\partial Q_s^{3,0}}{\partial e}\right]\sin(sl + 2h - 2n't)\ ; \qquad (34)$$

$$\delta h = -\frac{\gamma_{22}}{2W_1}\left(1+\frac{H}{G}\right)\left(\frac{L}{G}\right)Q_s^{3,2}\sin(sl + 2g + 2h - 2n't)$$

$$+\frac{\gamma_{22}}{2W_2}\left(1-\frac{H}{G}\right)\left(\frac{L}{G}\right)Q_{-s}^{3,2}\sin(sl - 2g + 2h - 2n't)$$

$$+\frac{\gamma_{22}}{W_3}\left(\frac{H}{L}\right)\left(\frac{L^2}{G^2}\right)Q_s^{3,0}\sin(sl + 2h - 2n't). \qquad (35)$$

CONCLUSION

This is a semi-analytical method of treating the influence of the ellipticity of the equator on the motion of a satellite. The method does not require a development of the disturbing function into powers of the eccentricity and, consequently, is valid for highly eccentric orbits. Its validity for orbits with a small eccentricity depends on the type of programming used. Programming with a floating decimal point would permit its use for such a case.

Long period terms will appear in the development of the perturbations if the mean daily motion of the satellite in question is commensurable with the angular velocity of the earth's rotation. The influence of the ellipticity of the earth's equator is greater for a satellite with a direct motion than for a satellite with a retrograde motion provided that e and i are small. This can easily be seen from an analytic development of the disturbing function, which shows that the order of the coefficients $Q_s^{p,q}$ of the long period terms with respect to e and $\sin i$ is higher if the motion is retrograde.

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